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# THE INFLUENCE OF ENDOGENOUS OFFER DURATIONS ON BARGAINING

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## ABSTRACT

The timing structure of a bargaining game is crucial. In the static bargaining model of Nash (1953), any division of the surplus is a possible equilibrium. In the alternating offer model of Rubinstein (1982), there is a unique subgame perfect Nash equilibrium (SPNE). This paper take a step towards endogenizing the timing of offers by expanding the strategy space to include the duration of the offer and permitting simultaneous moves. We find that virtually any division is a possible SPNE, even when one player is allowed to move first, and even when a player can refuse to receive an offer.

## 1. Introduction.

While Nash's (1953) cooperative bargaining solution is a seminal contribution, he believed it desireable to have a non-cooperative bargaining theory. It turns out that any division of total surplus is a possible Nash Equilibrium (NE) outcome of Nash's static non-cooperative bargaining game. Even repeated play and the refinement of subgame perfection does not shrink the set of possible outcomes.[1] Against this background, Rubinstein (1982) found a unique Subgame Perfect Nash Equilibrium (SPNE) of an alternating offer game.

There are a number of crucial assumptions in Rubinstein's model (e.g. perfect information, preference structure, and common knowledge), but this paper will focus on the timing structure. One player is arbitrarily picked to move first. He makes an offer. Then the second player can accept or reject this offer. If she rejects, then his offer expires, and she makes a counter offer, and so on, until some offer is accepted. No player can commit to an offer that extends for more than one period, and no player can make an offer at the same time as the other player.

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[1]The static game of dividing the dollar has each player simultaneously proposing a division. Iff the proposals agree, the dollar is accordingly divided. In the repeated version, the game ends iff the proposals agree; otherwise, old offers expire, and the game continues to a new round of proposals.

With discount factors less than unity, the first mover always has an advantage: i.e. he obtains more than if he were the second mover. Thus, both players would want to be the first mover given the choice. In other words, the alternating offer structure appears to be a binding constraint. Does the unique bargaining outcome hinge on this exogenously imposed structure? What if that structure were not imposed a priori on the bargaining process? What structure would arise endogenously?

To permit the endogenous determination of the timing structure, simultaneous offers are permitted and the strategy space is expanded to include the duration of an offer.[2] For example, you can offer an amount x and that offer stands until time t, except that you may alter x in the direction favorable to the other player, but not the other way. In other words, you grant the other player the right (until time t) to accept your most favorable outstanding offer. Further, before time t arrives, you may extend the offer to t' > t. While you are committed to your offers, this commitment does not prevent you from accepting the other player's offer during the interim. In the event that both players have outstanding offers and both accept each other's offers, the outcome is defined to be the midpoint between the offers.

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[2]Admati and Perry (1986) allow players to choose the duration of their offers but only within an alternating offer structure.

To keep the analysis tractable, time is assumed to come in discrete intervals (fixed but possibly very short - bounded by the minimum quantum of time necessary for communication). As in Rubinstein, the players bargain over the division of the [0,1] interval.

By appropriate choice of units, let the (0,1] interval denote the basic time interval. A duration of $\tau = 1/2$ is interpreted as an instant, so the other player does not have the right of acceptance. For example, if both players choose $\tau = 1/2$, then resolution occurs iff both players have proposed the same division; otherwise, the game continues to the next period. A duration of $\tau = 1$ is interpreted as granting the other player one opportunity to accept. A duration of $\tau = 3/2$ is interpreted as granting the other player one opportunity to accept and a commitment to an offer at least as favorable in the next period but perhaps only for an instant. Offers can be made or amended only at the beginning of each time period (i.e. at integer values of calendar time t).

As an intermediate step in analyzing the full influence of the time duration of offers, it is useful to consider a much simplier structure that will serve as a building block for the full model. In this simple structure all offers are required to have a duration of exactly one period; that is, each player has the right of acceptance. In the first stage, the players

simultaneously choose their offer amounts, and in the second stage, they simultaneously choose whether to accept or reject. An acceptance by either player ends the game. If both players reject, the game continues in the same form. Let's call this the XAR game (offers X followed by Accept or Reject). Section 2 shows that the set of SPNE outcomes of the XAR game is any division of [0,1].

Section 3 analyzes the game when the space of offer durations is {1/2, 1}. It is shown that the set of SPNE outcomes is again any division of [0,1]. Since the alternating offer structure is a feasible choice for the players in this game[3], the results of Section 3 show that Rubinstein's uniqueness result is a non-robust artifact of the exogenously imposed structure.

Section 4 analyzes the game with full timing flexibility. It is shown that the set of SPNE outcomes is again any division of [0,1]. Heuristically, since being a second mover is disadvantageous, each player will want to grant the other player the opportunity to accept his offer. The resulting simultaneity generates the continuum of SPNE outcomes.

To determine if the "any division is possible" result is

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[3]The period during which say player 1 would abstain from making an offer in Rubinstein's model can be represented in our model by interwoven time durations of 1/2 ,1, 1/2, 1, etc.

driven by the exogenous imposition of initial simultaneous moves, in Section 5 the model is modified by starting with a random choice of one player as a first mover. Thereafter, the strategy space for each player is as in Section 4. Note that again the alternating offer structure is feasible but not imposed. It turns out that for discount factors sufficiently close to unity, any division of (Ø,1) is possible.

Another way the players might be able to affect the timing structure is by refusing to receive (or listen to) an offer from the other player. For example, if bargaining is conducted through third party agents, a player could simply instruct his agent not to communicate the offers made by the other player except at specified times. Admanti and Perry (1986) suggest that such options might lead to an alternaing offer structure endogenously. Section 6 shows that such options do not alter the results of this paper - that any division is possible.

The implications of these results are discussed in Section 7. Tedious derivations are relegated to an Appendix.

## 2. The XAR Game.

After describing the detail of the XAR game, the construction of the SPNE is divided into three steps: (i) the

accept/reject (AR) subgame, (2) the best offer responses, and (3) the overall SPNE.

Since the players are bargaining over the division of $[0,1]$, it is convenient to express each player's offer in terms of the payoff to player 1. Let $(x_1,x_2)$ denote the respective offers. Then $x_i$ is interpreted as the proposed payoff to player 1 (whether i = 1 or 2).

Figure 1 depicts the game tree. Each player simultaneously chooses an offer. If $x_1 = x_2 = x$, the game automatically ends and the payoffs are $(x,1-x)$.[4] On the other hand, if $x_1 \neq x_2$, then each player has the right to accept the other player's offer. Should both accept, the game ends and the payoffs are $(m,1-m)$ where $m \equiv (x_1+x_2)/2$. If only one player accepts (say i), then the game ends and the payoffs are $(x_j,1-x_j)$. Finally, if neither player accepts, the game continues in the same form (denoted by "start over" in Figure 1). The payoffs from the next round (if any) are discounted by factors $(\delta_1,\delta_2)$ respectively. It is assumed that $\delta_i < 1$ for both players. For example, let $r_i$ be a positive rate of time preference and define $\delta_i \equiv \exp(-r_i u)$ where u is the length of

[4]Alternatively, one could allow the players to accept or reject when $x_1 = x_2$, since in the equilibrium constructed both choose to accept. On the other hand, in later sections when we consider offers that have a duration of 1/2, players do not have the right of acceptance, so we would have to amend the rule to be as stated in the above text. Our choice avoids rule changes and is w.l.o.g.

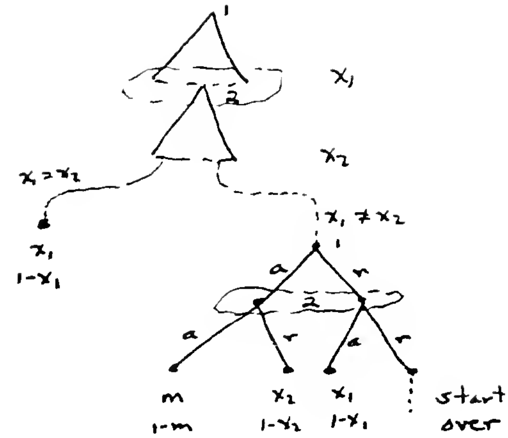



Figure 1

(2)

| (1) | accept | reject |
|---|---|---|
| a | $m, \quad 1-m$ | $x_2, \quad 1-x_2$ |
| r | $x_1, \quad 1-x_1$ | $\delta_1 v_1, \quad \delta_2 v_2$ |

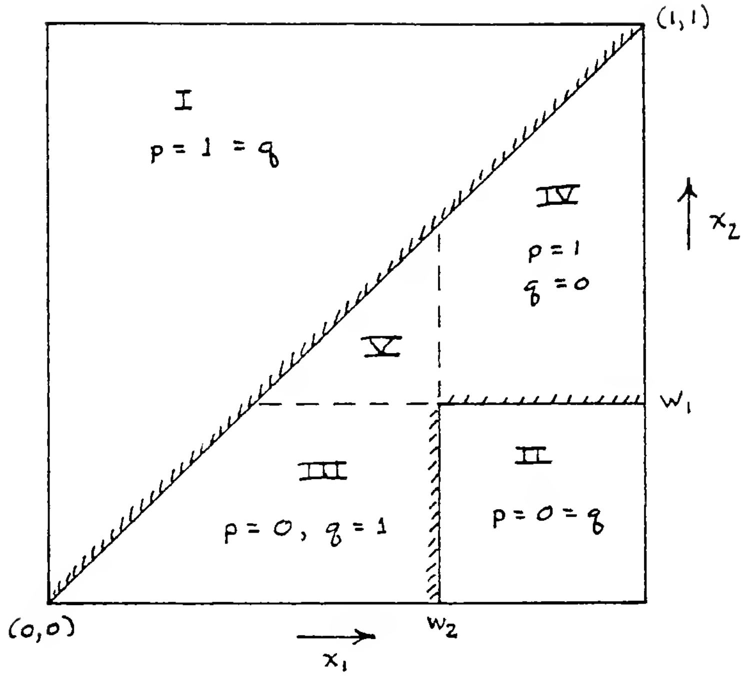
Figure 2

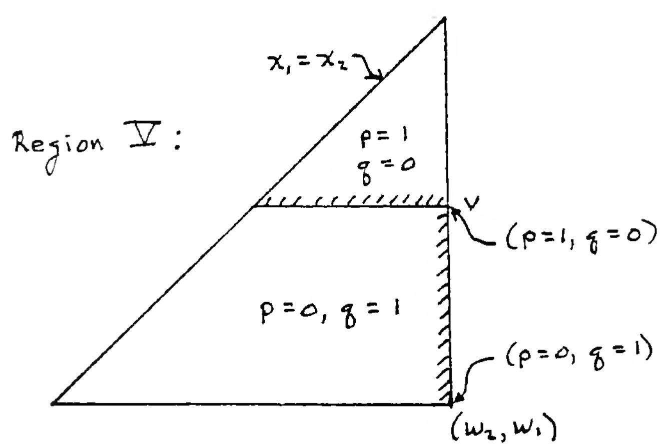
the time interval in the units of $r_1$.

Let $(v_1,v_2)$ be a possible SPNE outcome (i.e. the expected value of the payoff to each player). In the event that the game continues beyond the first period, note that at the start of the second period, the game tree from then on is identical to the original game tree. Therefore, $(\delta_1 v_1,\delta_2 v_2)$ is a possible SPNE expected payoff of the continuation game. Hence, we can analyze the right branch of the game tree in its normal form as depicted in Figure 2. We will confine our attention to SPNE outcomes that are "efficient": i.e. $v_1 + v_2 = 1$. With this focus, we can henceforth express the outcomes in terms of one parameter v, so $v_1 = v$ and $v_2 = 1-v$.

Given the offers $(x_1,x_2)$, it is elementary to compute all the NE of this AR subgame. For convenience of notation let $w_1 \equiv \delta_1 v$, and let $w_2 \equiv 1 - \delta_2(1-v)$. Note that $w_1 < w_2$ with strict inequality because $\delta_1 < 1$. To express mixed strategies, let $(p,q)$ denote the probability of acceptance by player 1 and 2 respectively.

For an arbitrary permissible value of $(w_1,w_2)$, Figure 3 shows one of the possible NE of the AR subgame for any value of $(x_1,x_2)$ in the unit square. The hash marks on the lines dividing two regions with different $(p,q)$ solutions indicate which solution is associated with that line. For example, $p=1=q$ along the diagonal. [These results are derived in the


(1,1)
I
p = 1 = q
IV
p = 1
q = 0
$x_2$
V
$w_1$
III
p = 0, q = 1
II
p = 0 = q
(0,0)
$w_2$
$x_1$


$x_1 = x_2$
Region V:
p = 1
q = 0
v
(p = 1, q = 0)
p = 0, q = 1
(p = 0, q = 1)
$(w_2, w_1)$

"E(v)"

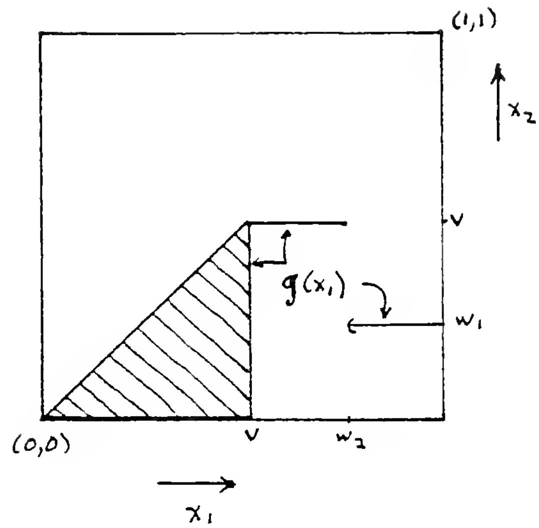
Figure 3.

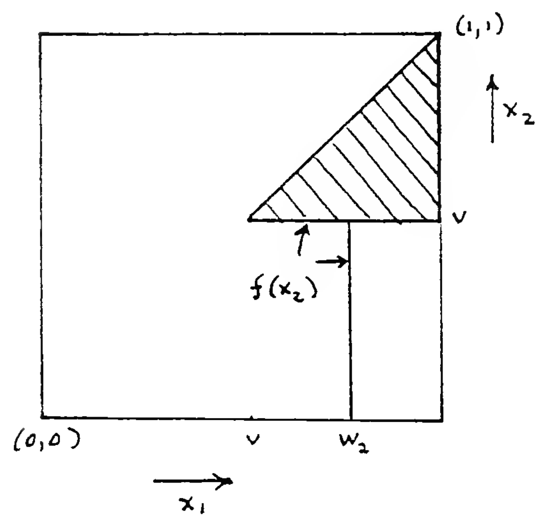
Appendix, but they are easy to verify directly.] There are uncountably many possible NE of the AR subgame. However, for our purposes, we will use only the NE depicted by Figure 3. Let E(v) denote this subgame NE. Note that E(v) involves only pure strategies. Also note that since we have specified the NE along the diagonal to have both players accept, the left branch of the game tree (Figure 1) is superfluous.

Having specified the NE of the AR subgame, the next step is to find the optimal choice of $(x_1,x_2)$. Using the NE payoffs just specified from the AR subgame, it is straight forward to compute the best offer correspondences of each player. Let $f(x_2)$ and $g(x_1)$ denote these correspondences for player 1 and 2 respectively. These best offer correspondences are plotted in Figure 4. [The derivation of expressions for f() and g() is given in the Appendix.] Overlaying the best response correspondences of Figure 4, one can see that their intersection is the line segment with $x_2 = v$, and $v \leq x_1 \leq w_2$.

The natural focal choice is $x_1 = x_2 = v$. Indeed, for any $v \in [0,1]$, these offers coupled with E(v) for the AR subgame clearly yield the payoff vector $(v,1-v)$, and thus constitute a consistent SPNE. We have, therefore, established:

Proposition 1. The set of SPNE outcomes of the XAR game includes $\{(v,1-v) \mid v \in [0,1]\}$.




Best Offer Correspondences

Figure 4.

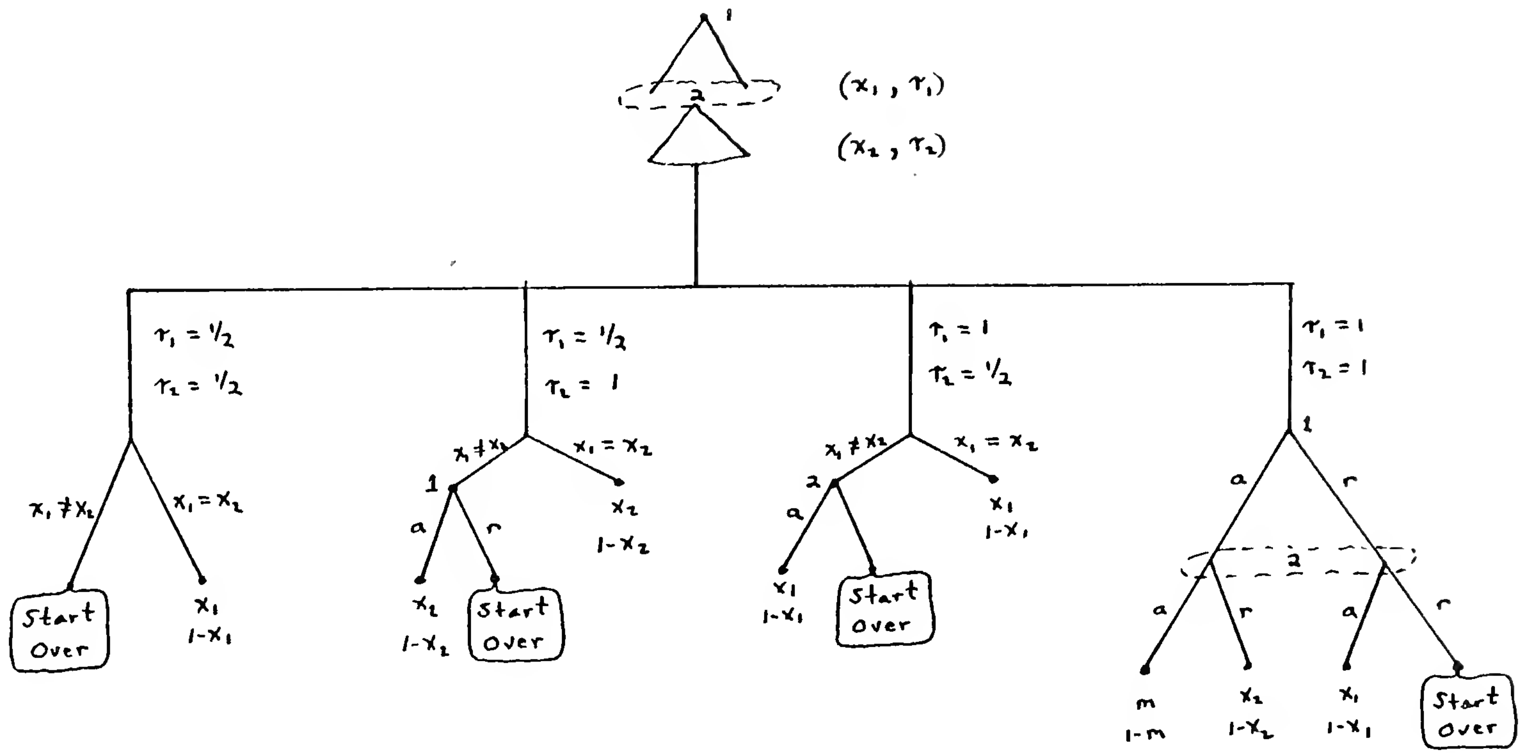
There are many other possible SPNE strategies which give the same set of outcomes (and possibly inefficient outcomes in addition), but they are beside the point, which is that <u>any division is a possible SPNE outcome.</u>

## 3. <u>The Choice of Granting the Right of Acceptance</u>.

To investigate the influence of the endogenous choice of offer duration, we next consider a model in which the duration can be $\tau_1 = 1/2$ or 1. Recall that $\tau_1 = 1$ means that player 1's offer is good for one period so player 2 has the opportunity to accept or reject. Recall that $\tau_1 = 1/2$ means that player 1's offer is good for an instant only. If it happens that $x_1 = x_2$ in this instant, the game ends; however, if the offers do not agree, then there is no opportunity for player 2 to accept $x_1$.

Let XAR(1/2,1) denote this game. The game tree is shown in Figure 5. As in Section 2, let (v,1-v) be the hypothesized expected payoff from a SPNE of the whole game. If at any point in the game the continuation game has the same form as the original game (indicated by "start over" in Figure 5), then we can replace the continuation game by a terminal node with undiscounted payoffs (v,1-v).

We will construct a SPNE whose equilibrium path involves only the fourth branch of the game tree and attains any




Offer-Duration Game with $\tau \in \{1/2, 1\}$

Figure 5.

division. Essentially the claim is that the additional option of choosing $\tau_1 = 1/2$ does not affect the SPNE property of the solutions constructed in Section 2.

Besides the fourth branch of the tree and the nodes where the game "starts over", there are two other nodes defining a subgame: (1) when $\tau_1 = 1/2$, $\tau_2 = 1$, and $x_1 = x_2$; and (2) the mirror image of case (1). For case (1), player 1 has an accept/reject decision. If v is the expected value of the continuation game, then clearly an optimal decision is to accept iff $x_2 \geq \delta_1 v \equiv w_1$. Analogously for case (2), player 2 should accept iff $x_1 \leq 1-\delta_2(1-v) \equiv w_2$. These subgame strategies are henceforth incorporated into the SPNE strategies.

Let's suppose that $(v,1-v)$ is a SPNE outcome that will be attained in every continuation game. It is proposed that the SPNE offers are $x_1 = x_2 = v$, and the SPNE durations are $\tau_i = 1$ for both. Whenever both players have the opportunity to accept, they play equilibrium E(v) [Figure 3], and whenever only one player has the opportunity to accept that player decides optimally as derived above. If the game continues beyond period one, these moves are repeated.

These strategies force the game along the fourth branch $(\tau_1 = 1 = \tau_2)$ which is the XAR game considered in Section 2. By the results of that section, we can conclude that the offers

are optimal for both (conditional on the choice of durations), and that the strategies are consistent with a SPNE outcome of $(v,1-v)$.

Now suppose player 1 deviates by choosing $\tau_1 = 1/2$, which forces the game down the second branch of the tree. We still have $x_2 = v$ and $\tau_2 = 1$. Note that player 2 does not have an opportunity to move again until the second period. What is 1's best offer now? If $x_1 = x_2$, then the game ends and player 1 gets v, which is the no-deviation payoff. If $x_1 \neq x_2$, then player one can still accept 2's offer and get v again, or he can reject her offer and start the game over. In the latter event and given the hypothesized v, the discounted payoff to player 1 would be $\delta_1 v < v$. Hence, player 1 cannot do better by choosing a shorter duration. Therefore, the specified strategies constitute a SPNE, and we have established the following result as promised.

<u>Proposition 2</u>. The set of SPNE outcomes of the XAR{1/2,1} game includes $\{(v,1-v) \mid v \in [0,1]\}$.

One may wonder whether the first branch of the XAR{1/2,1} game tree is compatible with a SPNE path, since it is essentially the static game for which any division is also possible. A necessary condition for such a SPNE with payoffs $(v,1-v)$ is $x_1 = x_2 = v$. Suppose $v > \delta_1$, and consider the following deviation by player 2: $\tau_2 = 1$, and $x_2 = \delta_1 v' + \varepsilon$, where

$0 < \varepsilon < v-\delta_1 v'$, and where $(v',1-v')$ is the expected payoff of the continuation payoff down the second branch. Note that for all $v' \in [0,1]$, $\delta_1 v' < x_2 < v$. This deviation forces the play down the second branch. Since $x_2 = x_1$, player 1 has the opportunity to accept which he will because a payoff of $x_2$ now is better than a payoff of $v'$ in the next period. The payoff to 2 is $1-(\delta_1 v'+\varepsilon)$ which exceeds $1-v$ by our choice of $\varepsilon$. Hence, deviation is profitable. Applying a similar argument when $v < 1-\delta_2$ establishes that player 1 can profitably deviate. Therefore, these extreme payoffs (i.e. $v < 1-\delta_2$ or $v > \delta_1$) cannot be supported as SPNE of the first branch. However, other payoffs could be supported by appropriate choice of $v'$ for the continuation game along the other branches.

We also cannot rule out the second and third branches completely. The alternating offer structure can be represented by alternating durations $(\tau_1,\tau_2)$ between $(1/2,1)$ and $(1,1/2)$. Rubinstein's solutions are possible SPNE paths involving the second or third branch. For instance, suppose initially $\tau_1 = 1/2$ and $\tau_2 = 1$, alternating thereafter; and the offers are $x_1 = 1-\delta_2(1-v')$ and $x_2 = \delta_1 v'$ where $v' = (1-\delta_2)/(1-\delta_1\delta_2)$ is the value of the continuation game. To sustain this path as a SPNE we can specify that $E(\delta_1 v')$ will be played in the event player 1 deviates by choosing $\tau_1 = 1$.

4. <u>General Offer-Duration Flexibility</u>.

Up to this point we have imposed three restrictions on strategies that we want to remove: namely (i) that outstanding offer amounts cannot be changed in any direction until after the offer expires, (ii) the offer duration cannot be changed, and (iii) the space of offer durations is {1/2,1}. As discussed in Section 1, it seems reasonable to permit a player to make his outstanding offer more favorable to the other player. It is only changes in a direction unfavorable to the other player that violate the spirit of good faith commitment.[5] It is also reasonable to permit a player to extend the duration of his outstanding offer before that offer expires, but to prohibit him from shortening the time duration. Finally, we will expand the space of durations in two steps. First, we will let offer durations be any positive integer. This simplification avoids half-integer durations (such as 3/2); as suggested by Section 3.2, these durations will not likely affect the possible SPNE payoffs. Let's call this game the "integer offer-duration game". The second step adds the half-integer durations to the strategy space, and analyzes this "general offer-duration game".

In this section it is convenient to express offers in

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[5]If a player (say 1) could increase his offer before the time duration arrives, then the time component looses its effect. Essentially, the commitment to the offer amount must be verified each instant, which is equivalent to having only instantaneous offers.

terms of the amount and the expiration date in calendar time rather than relative duration. Let calendar time be denoted by $t$, and let "period $t$" mean the interval $[t-1,t)$. An offer is a pair $\langle x_1, T_1 \rangle$ where $T_1$ is the expiration date. It is also a convenient simplification to initially rule out "pre-announcements": i.e. a statement at time $t$ committing to an offer which will not take effect until time $t+1$ or later.

## 4.1 The Integer Offer-Duration Game

The strategy space of offer durations is the set of positive integers. At the beginning of period $t$ (at time $t-1$), the relevant outstanding offers are those for which $T_1 \geq t$. If player 1 has an outstanding offer (say $y_1$), then he is constrained to honor this offer or improve it by making a new offer $\langle x_1, T_1 \rangle$ with $x_1 \leq y_1$ and $T_1 \geq t$. Similarly, if player 2 has an outstanding offer (say $y_2$), then she must honor it or improve it by making a new offer $\langle x_2, T_2 \rangle$ with $x_2 \geq y_2$ and $T_2 \geq t$. A player can also extend an offer before it expires. For example, at the beginning of period $t$, if $\langle y_1, U_1 \rangle$ is an outstanding offer (so $U_1 \geq t$), then $\langle y_1, T_1 \rangle$ with $T_1 > U_1$ is also a permissible offer modification. Old offers remain in effect until they expire or are extended.

The game proceeds as follows. First players choose offers simultaneously. Next they simultaneously decide to accept or

reject current outstanding offers. If both players accept, the payoffs are the midpoint of the two accepted offers. If only one player accepts, the payoffs are as prescribed by the accepted offer. If neither player accepts, the game proceeds to period two. At the beginning of period two, both players simultaneously make new and/or extend old offers. Next they simultaneously decide to accept or reject an offer among the currently outstanding offers, etc.

It should be clear that player 1 would never want to accept an outstanding offer whose amount is less than the maximum from among 2's current outstanding offers. Let $\tilde{x}_2(t)$ denote this maximum for period t. Similarly, player 2 would never want to accept an outstanding offer whose amount is more than the minimum from among 1's current outstanding offers; let $\tilde{x}_1(t)$ denote this minimum for period t. Thus, $\tilde{x}_i(t)$ gives the most favorable offer amount currently facing player i. Let $\tilde{T}_i(t)$ denote the expiration date of that most favorable offer. Then, $\langle\tilde{x}_i(t),\tilde{T}_i(t)\rangle$ is the most favorable offer currently facing player i.

Since there are two types of decisions, there are two corresponding types of nodes in the game tree. The first is the offer making decision, and the second is the accept/reject (AR) decision. Given offers $(\tilde{x}_1,\tilde{x}_2)$ and an expected payoff vector $(\nu,1-\nu)$ for the continuation game after AR, the possible NE AR strategies are the same as specified in Section 2. We

adopt solution configuration $E(\nu)$ applied to $(\tilde{x}_1,\tilde{x}_2)$. The particular $\nu$ will depend on the current action, and hence will be specified in the proof of Proposition 3 below for each potential deviation from equilibrium play.

The offer making nodes can be further categorized by the status of the outstanding offers: no outstanding offers, an outstanding offer by only one player, or outstanding offers by both players. On the other hand, considerable simplification and no loss of generality occurs if we always interpret the absence of an outstanding offer as a non-serious offer. In other words, for player 1 to have an outstanding offer of $\langle 1,t\rangle$ or to have no outstanding offer are strategically equivalent. Similarly, for player 2 to have an outstanding offer of $\langle 0,t\rangle$ or to have no outstanding offer is strategically equivalent. With this convention, we can treat every offer making node as if both players have outstanding offers.

The following strategies [coupled with $E(\nu)$ for the AR subgames] are proposed as SPNE strategies:

(1) If $\tilde{x}_2(t) \le v \le \tilde{x}_1(t)$, $\langle x_1,T_1\rangle = \langle v,t\rangle$ for both.

(2) If $\tilde{x}_1(t) \le \tilde{x}_2(t)$, $\langle x_1,T_1\rangle = \langle \tilde{x}_1(t),\tilde{T}_1(t)\rangle$ for both.

(3) If $\tilde{x}_1(t) > \tilde{x}_2(t) > v$, $\langle x_1,T_1\rangle = \langle \tilde{x}_2(t),t\rangle$ and

$\langle x_2,T_2\rangle = \langle \tilde{x}_2(t),\tilde{T}_2(t)\rangle$.

(4) If $v > \tilde{x}_1(t) > \tilde{x}_2(t)$, $\langle x_1,T_1\rangle = \langle \tilde{x}_1(t),\tilde{T}_1(t)\rangle$ and

$\langle x_2,T_2\rangle = \langle \tilde{x}_1(t),t\rangle$.

These strategies produce the same equilibrium path as the simple XAR game of Section 2. That is, both players offer $\langle v,1\rangle$, which both accept in the first period, and the game ends. If any other path is taken (by mistake), essentially the players correct outstanding offers that are unattractive and take advantage of offers that are unexpectedly attractive.

By proving for any $v \in [0,1]$ that the proposed strategies constitute a SPNE, we will have established the following main result.

<u>Proposition 3</u>. The set of SPNE outcomes of the integer offer-duration game includes $\{(v,1-v) \mid v \in [0,1]\}$.

**PROOF:** [On the first pass, the reader may skip this lengthy proof and go directly to Section 4.2.] Hypothesize that the expected payoff vector of the game is $(v,1-v)$ for some $v \in [0,1]$. Suppose player 1 adopts the proposed strategies at all nodes of the game tree. First, we will show that starting from the initial node, player 2 can do no better than to adopt the proposed strategy. Second, we will show that starting from any other node, player 2 can do no better than to adopt the proposed strategy.

(1) At the initial node the proposed strategy has $\langle x_i,T_i\rangle = \langle v,1\rangle$ for both players. Consider deviations by player 2. First, consider a deviation in duration alone ($T_2 > 1$). Since

$x_1 = x_2 = v$, in the first period independent of the expected continuation payoffs, accept/accept is an AR NE. Thus, the game will end in period one with payoffs $(v,1-v)$. Second, consider a deviation with $x_2 > v$. Independent of the expected payoffs for the continuation game, the AR decision is accept/accept, ending the game in period one with payoffs $(m',1-m')$ where $m' = (x_2+v)/2$, which is strictly worse for player 2. Last, consider a deviation with $x_2 < v$ and $T_2 \geq 1$. The AR outcome depends on the expected payoffs $(\nu,1-\nu)$ in the continuation game. If $\nu \geq v$, then reject/accept (meaning $p=0$, $q=1$) is the AR outcome which ends the game in period one with payoffs $(v,1-v)$. Thus, it will suffice to show that $\nu \geq v$ is always a possible outcome of the continuation game. We will demonstrate this in two steps.

(a) If $T_2 = 1$, then the continuation game starting in period two is equivalent to the original game, so by hypothesis $\nu = v$ is a possible NE outcome.

(b) If $T_2 > 1$, then player 1 offers $\langle v,1\rangle$ as long as $\tilde{x}_2(t) < v$. Suppose that after a (possibly zero) number of extensions or improvements with $\tilde{x}_2(t) < v$, player 2 offers $\langle y_2,t\rangle$ with $y_2 \geq v$ and $t > 1$. Clearly, in period t, the AR solution is accept/accept (independent of the expected continuation payoffs), ending the game with payoffs $(m'',1-m'')$ where $m'' = (y_2+v)/2 \geq v$. Thus, $\nu \geq v$ is a possible NE outcome for period t-1. Then in period t-1, reject/accept is a NE yielding payoffs $(v,1-v)$. By backward induction reject/accept is a NE for every previous period including

period 1 yielding $(v,1-v)$. This result also holds in the limit as the expiration date $\tilde{T}_2(t)$ goes to infinity.

Hence, player 2 can do no better by deviating from the proposed strategy starting at the initial node.

(2) Suppose (presumably by mistake) the game continues beyond the first period. We consider four cases.

(a) Suppose $\tilde{x}_2(t) \leq v \leq \tilde{x}_1(t)$. Player 1 will make a new offer $\langle v,t \rangle$. By the argument of (1), $(v,1-v)$ is a consistent continuation payoff. Then clearly player 2 can do no better than offer $\langle v,t \rangle$, ending the game with payoffs $(v,1-v)$.

(b) Suppose $\tilde{x}_1(t) \leq \tilde{x}_2(t)$. Because player 1 can only lower $x_1$ and player 2 can only raise $x_2$, then for all $t' > t$, $\tilde{x}_1(t') \leq \tilde{x}_2(t')$. Therefore, independent of the expected continuation payoffs, accept/accept is a NE, ending the game immediately with player 2 getting $1-(\tilde{x}_1(t)+x_2)/2$. Clearly, player 2 can do no better than to let her offer stand at $\tilde{x}_2(t)$.

(c) Suppose $\tilde{x}_1(t) > \tilde{x}_2(t) > v$. The prescribed strategy has player 1 making a new offer $\langle \tilde{x}_2(t),t \rangle$. Player 2 is constrained by $x_2 \geq \tilde{x}_2(t)$. Therefore, independent of the value of the continuation game, the AR solution is accept/accept, ending the game in period t with payoffs $(\tilde{x}_2,1-\tilde{x}_2)$. Therefore, player 2 can do no better than $x_2 = \tilde{x}_2(t)$ as prescribed.

(d) Suppose $v > \tilde{x}_1(t) > \tilde{x}_2(t)$. Player 1 is constrained

by $x_1 \geq \tilde{x}_1(t)$, and the prescribed strategy is to offer $\langle \tilde{x}_1(t), \tilde{T}_1(t) \rangle$. The prescribed strategy for player 2 is to offer $\langle \tilde{x}_1(t), t \rangle$, which would result in immediate AR solution accept/accept and payoffs $(\tilde{x}_1, 1-\tilde{x}_1)$. Clearly, $x_2 > \tilde{x}_1(t)$ is suboptimal. For any $\tilde{x}_2(t) \leq x_2 \leq \tilde{x}_1(t)$, if $\nu \geq \tilde{x}_1(t)$, then the current period AR solution is reject/accept ending the game with payoffs $(\tilde{x}_1, 1-\tilde{x}_1)$. Therefore, conditional on showing that $\nu \geq \tilde{x}_1(t)$ is a possible NE outcome for the continuation game, player 2 cannot do better than with the prescribed strategy. Consider two subcases.

(i) Suppose the game continues in disagreement until time $\tilde{T}_1(t) \leq \tilde{T}_2(t)$. By our convention, in the next period $\tilde{x}_1 = 1 \geq v > \tilde{x}_2$, which is case (2)(a), so $(v, 1-v)$ is a consistent expected payoff. Then by backward induction $\nu \geq \tilde{x}_1(t)$ for period t as hypothesized.

(ii) Suppose the game continues in disagreement until $\tilde{T}_1(t) > \tilde{T}_2(t)$. After period $\tilde{T}_1(t)$, player 1 will offer v. Clearly, it is not optimal for player 2 to have an outstanding offer at this time that is greater than v, and since $\tilde{x}_2 < v$, she can ensure that this does not happen. Then by case (2)(a), $(v, 1-v)$ will be the payoff in period $\tilde{T}_1(t)+1$. In period $\tilde{T}_1(t)$, player 2 will never want to offer more than $\tilde{x}_1(t)$; and any offer $x_2 \leq \tilde{x}_1(t) < v$ will result in reject/accept yielding payoffs $(\tilde{x}_1, 1-\tilde{x}_1)$. By backward induction $\nu = \tilde{x}_1(t)$ holds back to period t.

That player 1 can do no better given player 2 adopts the proposed strategy can be shown by a symmetric argument. Q.E.D.

## 4.2 The Half-Integer Offer-Duration Game.

Consider the offer-duration game of Section 4.1 with the space of durations expanded to the set {K/2 | K is a positive integer. Recall that the half-integer durations mean that an offer is made which is good if and only if it is instanteously matched; otherwise the other player does not have the right to accept or reject the offer. The results of Section 3 suggest that this extension should not change the set of SPNE outcomes.

The offer making nodes of the game tree are identical to those of the integer game. The AR decision nodes which occur at whole integer times are also identical to those of the integer game. The additional decision nodes are all "asymmetric AR" nodes: e.g. the only outstanding offer of player 1 has a duration of $\tau$ = 1/2, so player 2 has no opportunity to accept, but player 1 has the opportunity to accept player 2's offer. Note that both $\tau_i$ = 1/2 is a possible node, but then the players have no decision to make; the game ends iff the offers match. Intuitively, bargaining power comes from making an offer the other player cannot rationally refuse; thus, players cannot gain from making offers without the right of acceptance.

To the strategies specified in Section 4.1, we must

add strategies for the asymmetric AR decision nodes. Let $\bar{T}_i$ denote the maximum duration of player i's outstanding offers in a given period. Let $(\nu, 1-\nu)$ be the expected payoffs of the continuation game after an asymmetric AR period.

(5) (a) $\underline{\bar{T}_1 = 1/2 < \bar{T}_2}$. Player 1 accepts iff $x_2 \geq \delta_1 \nu$.

(b) $\underline{\bar{T}_2 = 1/2 < \bar{T}_1}$. Player 2 accepts iff $x_1 \leq 1-\delta_2(1-\nu)$.

Clearly, by construction, these strategies are optimal. Let (v,1-v) be an hypothesized expected payoffs of this general offer-duration game. We want to show that for any $v \in [0,1]$, the proposed strategies (1)-(5) [coupled with $E(\nu)$] constitute a SPNE. We will then have:

<u>Proposition 4</u>. The set of SPNE outcomes of the general offer-duration game includes $\{(v,1-v) \mid v \in [0,1]\}$.

**PROOF:** Suppose that player 1 adheres to the proposed strategies. We want to show that player 2 never has an incentive to make an offer resulting in $\bar{T}_2 = 1/2$. Such a result can occur in only two ways: (a) player 2 has no outstanding offers at the start of period t and chooses $\langle x_2, t-1/2 \rangle$; (b) the only outstanding offer by player 2 for period t has a duration of 1/2. In case (a), the possibility of making the offer in question implies that player 2 is not constrained in the amount of the offer. Therefore, player 2 could offer $\langle 0, t \rangle$ (which player 1 will reject) and then accept player 1's

offer giving player 2 the same payoff as would the offer with a duration of 1/2. In case (b), there must have been a prior period (say t-k) in which player 2 made an offer with expiration date t-1/2. An alternative feasible action was to shorten the expiration date to t-1 and plan to offer <0,t> for period t. As in case (a), this plan would give the same expected payoff, so there is no advantage to choosing half-integer expiration dates. Q.E.D.

### 4.3 Robustness to Pre-Announcements.

Throughout Section 4, we have not allowed the players to announce at time t an offer that will not take effect until some period after the current one. Now we will argue that this restriction is inconsequential to the results. The hypothesis is that the strategies of Section 4.2 still constitute a SPNE.

First, consider a pre-announced offer that is less favorable than currently outstanding offers. Since it is common knowledge that players are allowed to make their offers more favorable, the threat implied by the pre-announcement is not credible, and hence will be ignored.

Secondly, consider a pre-announced offer that is more favorable than currently outstanding offers. How could such a pre-announcement help a player (say 2)? If it induces player 1

to wait for that offer to take effect, then both bear delay costs implying that player 2 gets less than if she made the more favorable offer now. Since she is not prevented from making more favorable offers now, such a pre-announcement cannot be optimal.

Finally, notice that pre-announced offers that are equal to the best current outstanding offer are superfluous. As a threat that future offers will not be more favorable, they are not credible. On the other hand, they will not induce delay. Therefore, the flexibility to make pre-announcements is inconsequential to the results.

5. Effect of an Exogenous First Mover.

In all the bargaining games considered so far, we have both (a) removed the exogenous imposition of the alternating offer structure, and (b) exogenously imposed simultaneous moves from the start of the game. The purpose of this section is to demonstrate that the absence of a first mover is not crucial.

Suppose one player is exogenously selected as the first mover (say player 1). In "period zero" he chooses an offer amount and duration. Next, player 2 decides whether to accept or reject 1's offer (presuming the right of acceptance is granted). If accepted, the game ends. Otherwise, the

continuation game is a general offer-duration game (with perhaps an outstanding offer by player 1).

Let the expected payoff vector of the game from period one on be $(\nu, 1-\nu)$. Then, clearly it is optimal for player 2 to accept iff $x_1 \leq 1-\delta_2(1-\nu)$. Hence, the optimal offer amount by player 1 is $x_1 = 1-\delta_2(1-\nu)$. I claim the optimal offer duration by player 1 in period zero is one period. The proposed SPNE strategies are those specified in Section 4, plus

(6)(a) Player 1 offers $\langle 1-\delta_2(1-\nu), 1\rangle$.

(b) Player 2 accepts iff $x_1 \leq 1-\delta_2(1-\nu)$.

<u>Proposition 5</u>. The set of SPNE outcomes of the general offer-duration game with player 1 as the first mover includes $\{(v, 1-v) \mid v \in [1-\delta_2, 1]\}$. Moreover, as the time period between offers shrinks to zero, $\delta_2$ goes to 1, so any division in $(0,1]$ becomes possible.

**PROOF:** First observe that if player 1 adopts the strategy of (6)(a), specifically a duration of one period, then the continuation game is precisely the general offer-duration game of Section 4; therefore, any $\nu \in [0,1]$ is possible. Thus, the payoff to player 1 can range from $1-\delta_2$ to 1 as claimed. It only remains to show that player 1 can do no better by choosing a duration greater than one period. If he chooses a longer duration, then the continuation game (if reached) begins with

player 1 having the only outstanding offer, say $\tilde{x}_1$. From Section 4, the expected payoff to player 1 can be any $v \leq \tilde{x}_1$. Clearly, $\tilde{x}_1 < \nu$ is a suboptimal deviation, so suppose $\tilde{x}_1 \geq \nu$. But then by selecting $v = \nu$, player 1 can do no better with a duration greater than one period. Q.E.D.

An analogous result obtains when player 2 is the first mover. Period zero strategies are

(6′)(a) Player 2 offers $\langle \delta_1 \nu, 1 \rangle$.

(b) Player 1 accepts iff $x_2 \geq \delta_1 \nu$.

The set of possible SPNE outcomes includes $\{(v,1-v) \mid v \in [0,\delta_1)\}$. Then, as the time period between offers shrinks to zero, any division in $[0,1)$ becomes possible.

There is a first-mover advantage, but only in the sense that the first mover can ensure a minimum payoff: $1-\delta_2$ for player 1, and $1-\delta_1$ for player 2. As the time period shrinks to zero, this first-mover advantage vanishes. Typically, there would be a large range of outcomes that depend on the common beliefs about the ultimate devision $(v,1-v)$.

## 6. <u>The Choice of Receiving Offers</u>.

One way for a player to attempt to have the only

outstanding offer and thereby to gain some bargaining power is to refuse to receive (or listen to) any offers by the other player except at specified times. To model this option, we expand the strategy space at each period to include the decision to listen (L) or not-to-listen (N). At the beginning of each period, both players make an offer and choose L or N. If both choose L, the payoffs are the same as before (depending on the offers and the AR decisions). If both choose N, the game continues into the next period without resolution. If one player, say i, chooses L and the other chooses N, the game ends if and only if player i accepts.[6]

Consider the XAR game of Section 2 modified by adding {L,N} to the strategy space as depicted in Figure 6. Both players simultaneously choose an offer $x_i$ and L or N. At the next node, if player 1 chose L, he knows that he is at branch C or D, and he knows both offers $x_1$ and $x_2$. On the other hand, if he chose N, he knows that he is at branch A or B, and he knows only his own offer $x_1$. Similarly, if player 2 chose L, she knows that she is at branch B or D, and she knows both

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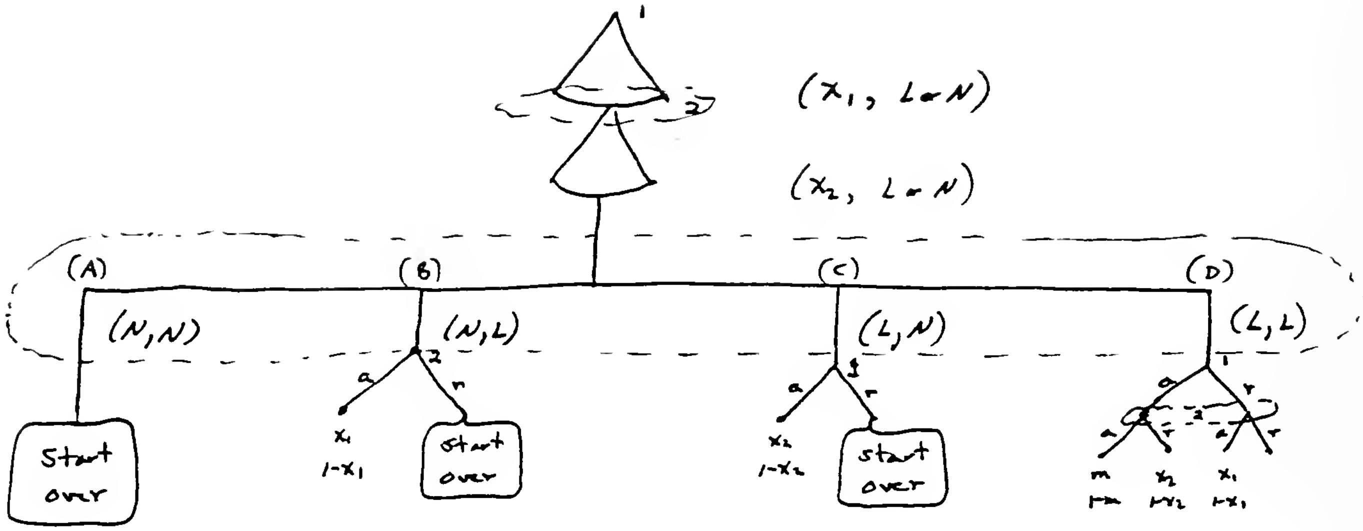
[6]Alternatively, we could imagine that player 2 gives instructions to her third party agent to accept certain offers. Observe, however, that this setup renders the choice of L/N irrelevant, since the actions by player 2 are in no way restricted by the choice of N. As with all games, player 2 can contemplate all possible offers by player 1 and generate instructions which can then be implemented by a third party; she need not personally hear the offer $x_1$. Thus, for the choice of N to be relevant, we must rule out acceptance instructions in the event that N is choosen. That is, if player 1 chooses L and 2 chooses N, and if 1 rejects $x_2$, then the game continues into the next period without resolution.




Player 1's information sets: $\{(C \cup D),\ x_1, x_2\}$ and $\{(A \cup B),\ x_1\}$

Player 2's information sets: $\{(B \cup D),\ x_1, x_2\}$ and $\{(A \cup C),\ x_2\}$

Choice of Receiving Offers

Figure 6.

offers. If instead she chose N, then she knows that she is at branch A or C, and she knows only her own offer $x_2$.

Since there are no proper subgames within one period, the appropriate equilibrium concept for this modified bargaining game is that of <u>sequential equilibrium</u> (SE) [Kreps and Wilson (1982)], which specifies a belief as well as a strategy for each player at each relevant information set. In particular, when player 1 is contemplating his accept/reject decision, does he believe player 2 has choosen L or N? Similarly, when player 2 is contemplating her accept/reject decision, does she believe player 1 has choosen L or N? The only a priori restriction on beliefs is that they must obey Bayes Rule.

Clearly when any player chooses N, he gains absolutely no information about which of the two possible branches he is at. Moreover, when any player chooses L, the added information of the other's offer also does not reveal which of the two possible branches he is at. Therefore, in this game, Bayes Rule puts no constraints on the up-dated beliefs.

We will specify that both players believe that the other has choosen L. Then, it is claimed that (L,L) coupled with $x_1 = x_2 = v$ and the AR solution E(v) is a SE. Clearly, the hypothesized SE payoffs are (v,1-v). Suppose player 1 adopts the proposed strategies, while player 2 deviates. We have shown in Section 2 that, given player 2 chooses L, there is no

profitable deviation in offer or AR decision. Now suppose player 2 deviates by choosing N. Since player 1 believes 2 has choosen L, he will continue to make the same AR decision, which is to accept iff $x_2 \geq v$ (given $x_1 = v$). Any $x_2 > v$ is worse for 2. Any $x_2 < v$ will be rejected, forcing the game to continue, and giving 2 an expected discounted payoff of $\delta_2(1-v) < (1-v)$. Hence, 2's best offer is $x_2 = v$, yielding $1-v$. Therefore, player 2 cannot profitably deviate. A similar argument holds for deviations by player 1.

We have therefore shown that <u>any $v \in [0,1]$ is a possible SE outcome</u> of the XAR game with the expanded {L,N} strategy space. It is left as an exercise for the skeptical reader to verify that the same result holds for the general offer-duration game of Section 4 with the added option of not listening. It is also worthwhile remarking that the above SE result survives the refinements of the "intuitive criteria" [e.g. Kreps (1984), Cho (1985)] because there is no offer that can separate a player who has chosen N from one who has chosen L.

## 7. <u>Summary and Discussion</u>.

When the duration of offers in a bargaining game are part of the players' strategy space and simultaneous offers are permitted (at least after the first move), then virtually any

division of the surplus is a possible SPNE outcome. Hence, the uniqueness result of Rubinstein (1982) for the alternating offer structure is an artifact of that exogenously imposed timing structure. The intuitive explanation is that given the choice no player wants to be without an outstanding offer the other player can't refuse [the "Godfather principle"]. Both players choose to move simultaneously. The power of subgame perfection to prune non-credible strategies is considerably weaken by simultaneous moves. As a consequence, the offer-duration game is effectively equivalent to Nash's static game.

While it is important to recognize the sensitivity of bargaining outcomes to the timing structure, it may also be somewhat disheartening that theory permits such a wide range of possible outcomes. How can one predict the outcome?

Reflecting on the proofs of our results, a very strong implicit presumption stands out: that the ultimate division $(\nu, 1-\nu)$ for every subgame is common knowledge. It is at least as strong a presumption as its counterpart in Nash's static game. How could it be that the players come to have this common knowledge? If we had a basis for discovering this common knowledge, then we would have a method for prediciitng bargaining outcomes of an offer-duration game. The work of Roth and Schoumaker (1983), is directly related to this approach. Their experiments provide evidence that prior beliefs about the expected division are indeed important

explanatory factors.

Another approach that comes to mind is to think of the beliefs about the division (v,1-v) as private information that may become revealed through the bargaining process. Cramton (1984) studied a trading model with two-sided uncertainty in which the time pattern of offers eventually revealed the private information. However, Cramton's work is not directly applicable to our offer-duration bargaining game for two important reasons. First, Cramton incorporates the alternating offer structure as the last phase of his game after both players have revealed their private information. Therefore, his results may be (like Rubinsteins's) quite sensitive to that modelling assumption.[7] Secondly, if the prior beliefs of the players differ, it is not clear what the implications are for the bargaining outcome if and when these beliefs become revealed. In Cramton's model, the private information is the value of an object to the players; revelation then defines the size of the pie to be divided. There is no apparent isomorphism between Cramton's model and a offer-duration game with diverse beliefs about (v,1-v).

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[7]Cramton also assumes that the limit of the Rubinstein solution as the time period shrinks to zero is the unique solution to the limit game in continuous time. However, as we have seen, the simultaneity implicit in continuous time bargaining introduces the possibility of a continuum of possible outcomes.

# APPENDIX

## Derivation of NE Strategies for AR Subgame (Figure 2)

The expected payoffs are

$$E\pi_1 = pqm + p(1-q)x_2 + (1-p)qx_1 + (1-p)(1-q)w_1 ,$$

$$E\pi_2 = pq(1-m) + p(1-q)(1-x_2) + (1-p)q(1-x_1) + (1-p)(1-q)(1-w_2) .$$

The relevant partial derivatives are given here:

$$\partial E\pi_1/\partial p = q(w_1 - m) + (x_2 - w_1)$$

$$\partial E\pi_2/\partial q = p(m - w_2) + (w_2 - x_1) .$$

Setting these partial derivatives to zero defines $\hat{p}$ and $\hat{q}$:

$$\hat{p} \equiv (w_2 - x_1)/(w_2 - m) ;$$

$$\hat{q} \equiv (w_1 - x_2)/(w_1 - m).$$

We develop the AR solutions for six cases (A) - (F).

(A) Suppose $x_2 < w_1$. Note that if $x_1 > x_2$, then $\partial E\pi_1/\partial p <$ $0$ for all values of $q$, so the optimal response is $p=0$. If $x_1$ $\leq x_2$, then $\tilde{q} \in [0,1]$, and $\partial E\pi_1/\partial p > (<)\ 0$ as $q > (<)\ \tilde{q}$. Hence, $p=1$ is optimal when $q > \tilde{q}$; $p=0$ is optimal when $q < \tilde{q}$; and any $p$ is optimal when $q = \tilde{q}$.

(B) Suppose $x_1 < w_2$. Note that if $x_1 < x_2$, then $\partial E\pi_2/\partial q >$ $0$ for all values of $p$, so the optimal response is $q=0$. If $x_1$ $\geq x_2$, then $\tilde{p} \in [0,1]$, and $\partial E\pi_2/\partial q > (<)\ 0$ as $p < (>)\ \tilde{p}$. Hence, $q=0$ is optimal when $p > \tilde{p}$; $q=1$ is optimal when $p < \tilde{p}$; and any $q$ is optimal when $p = \tilde{p}$.

(C) Suppose $x_2 = w_1$. Note that if $q=0$ or $w_1 = m$, then $\partial E\pi_1/\partial p = 0$, so any $p$ is optimal. On the other hand, if $q > 0$ and $w_1 > m$, then $p=1$ is optimal; and if $q > 0$ and $w_1 < m$, then $p=0$ is optimal.

(D) Suppose $x_1 = w_2$. Note that if $p=0$ or $w_2 = m$, then $\partial E\pi_2/\partial q = 0$, so any $q$ is optimal. On the other hand, if $p > 0$ and $w_2 > m$, then $q=0$ is optimal; and if $p > 0$ and $w_2 < m$, then $q=1$ is optimal.

(E) Suppose $x_2 > w_1$. Note that if $x_1 < x_2$, then $\partial E\pi_1/\partial p >$ $0$ for all values of $q$, so the optimal response is $p=1$. If $x_1$ $\geq x_2$, then $\tilde{q} \in [0,1]$, and $\partial E\pi_1/\partial p > (<)\ 0$ as $q < (>)\ \tilde{q}$. Hence, $p=1$ is optimal when $q < \tilde{q}$; $p=0$ is optimal when $q > \tilde{q}$; and any $p$

is optimal when $q = \hat{q}$.

(F) Suppose $x_1 < w_2$. Note that if $x_1 > x_2$, then $\partial E\pi_2/\partial q < 0$ for all values of $p$, so the optimal response is $q=0$. If $x_1 \leq x_2$, then $\hat{p} \in [0,1]$, and $\partial E\pi_2/\partial q > (<) \ 0$ as $p > (<) \ \hat{p}$. Hence, $q=1$ is optimal when $p > \hat{p}$; $q=0$ is optimal when $p < \hat{p}$; and any $q$ is optimal when $p = \hat{p}$.

Now let's consider the five regions of Figure 3. Recall that Z is defined as the point $(x_1,x_2) = (w_2,w_1)$.

<u>Region I ($x_1 \leq x_2$)</u>. Starting from the upper right-hand corner and moving to the left-hand corner, there are three relevant subregions and the diagonal line. The diagonal will be treated separately. (a) $w_2 < x_1$: from (E), $p=1$; then, since $p > \hat{p}$, from (F), $q=1$. (b) $x_1 < w_2$ and $x_2 > w_1$: from (B), $q=1$; and from (E), $p=1$. On the boundary between (a) and (b), where $x_1 = w_2$, from (E), $p=1$, so from (D) $q=1$. (c) $x_1 < w_1$: from (B), $q=1$; then, since $q > \hat{q}$, from (A), $p=1$. On the boundary between (b) and (c), where $x_2 = w_1$, from (B), $q=1$, so from (C) $p=1$.

Along the diagonal, note that $\hat{p} = 1 = \hat{q}$. Where the diagonal borders (a), from (E), $p=1$, and from (F) any $q$ is optimal including $q=1$. Where the diagonal borders (c), from (B) $q=1$, and from (C) any $p$ is optimal including $p=1$. At the points $(x_1,x_2) = (w_1,w_1)$ and $(w_2,w_2)$, there are continua of NE of the form $\{(p,1), (1,q)\}$ for any $p$ and $q$ values, which satisfy (B)(C) and (D)(E). Along the rest of the diagonal,

there are three distinct NE: $(p,q) \in \{(0,1), (1,0), (1,1)\}$, which satisfy (B) and (E). [In constructing a SPNE for the XAR game, we specify that both players accept along the diagonal; hence $(p=1=q)$ throughout this region.]

Region II ($x_1 > w_2$, $x_2 < w_1$, and Z). On the interior of this region, from (A) $p=0$, and from (F) $q=0$. At Z there is a continuum of NE of the form $\{(0,q), (p,0)\}$ satisfying (C) and (D). [In constructing a SPNE for the XAR game, we specify that $p=0$ and $q=1$ for point Z.]

Region III ($x_2 < x_1 \leq w_2$ and $x_2 \leq w_1$, less Z). On the interior of this region, from (A) $p=0$; and, since $p < \bar{p}$, from (B), $q=1$. On the right boundary ($x_1 = w_2$), (A) still implies $p=0$; hence, from (D), any q is optimal. On the top boundary ($x_2 = w_1$), $(0,1)$ satisfies (B) and (C). In addition, there is a continuum of NE along the top boundary of the form $(p,0)$ for $p \geq \bar{p}$, satisfying (B) and (C). [In constructing a SPNE for the XAR game, we specify that $p=0$ and $q=1$ for the right and top boundary.]

Region IV ($w_1 \leq x_2 < x_1$ and $x_1 \geq w_2$, less Z). On the interior of this region, from (F) $q=0$; and, since $q < \bar{q}$, from (E), $p=1$. On the bottom boundary ($x_2 = w_1$), (F) still implies $q=0$; hence, from (C), any p is optimal. [In constructing a SPNE for the XAR game, we specify that $p=1$ and $q=0$ for this bottom boundary.] On the left boundary ($x_1 = w_2$), $(1,0)$ satisfies (D)

and (E). In addition, there is a continuum of NE along the left boundary of the form $(0,q)$ for $q \geq \hat{q}$, satisfying (D) and (E). [In constructing a SPNE for the XAR game, we specify that $p=0$ and $q=1$ for this left boundary.]

<u>Region V ($w_1 < x_2 < x_1 < w_2$)</u>. (B) and (E) cover this region. It can be easily verified that $(p,q) = (0,1)$ and $(1,0)$ are both NE. Furthermore, $(\hat{p},\hat{q})$ is also a NE. [In constructing a SPNE for the XAR game, we specify that $p=1$ and $q=0$ if $v \leq x_2 < x_1$, and that $p=0$ and $q=1$ otherwise.]

## <u>Derivation of the Best Offer Correspondences</u>.

Given the solution configuration $E(v)$ for the AR subgame, the task is to derive the best offer correspondences $f(x_2)$ and $g(x_1)$, which are the basis for Figure 4. Let us take $g(x_1)$ first. Note that it is never optimal for player 2 to offer $x_2 > x_1$, because both would accept giving 2 a payoff of $1 - (x_1+x_2)/2$ which is strictly decreasing in $x_2$. Hence, below we consider only offers $x_2 \leq x_1$.

(1) $x_1 = 0$. Trivially, $g(0) = 0$.

(2) $0 < x_1 \leq v$. Any $x_2 < x_1$ will be accepted by player 2 alone; hence $E\pi_2 = 1-x_1$, which is the same as when $x_2 = x_1$. Therefore, $g(x_1) = [0,x_1]$.

(3) $v < x_1 \leq w_2$. Any offer $v \leq x_2 \leq x_1$ will be accepted

by player 1 alone; hence $E\pi_2 = 1-x_2$, so $x_2 > v$ is not optimal. Any $x_2 < v$ will be accepted by player 2 alone; hence $E\pi_2 = 1-x_1 < 1-v$. The offer $x_2 = v$ will be accepted by player 1 alone yielding 1-v for player 2. Therefore $g(x_1) = v$.

(4) $w_2 < x_1 \leq 1$. Any offer $x_2 < w_1$ will be rejected by both players yielding player 2 an expected payoff for the continuation game of $\delta_2(1-v) = 1-w_2$. Any offer with $w_1 \leq x_2 < x_1$ will be accepted by player 1 alone giving player 2 payoff $1-x_2$, which is decreasing in $x_2$; hence, $x_2 > w_1$ is not optimal. For $x_2 = w_1$, $E\pi_2 = 1-w_1 > 1-w_2$. Therefore, $g(x_1) = w_1$.

A parallel derivation can be obtained for $f(x_2)$. There are four similar relevant ranges.

(1′) $f(1) = 1$.

(2′) For $v \leq x_2 < 1$, $f(x_2) = [x_2, 1]$.

(3′) For $w_1 < x_2 < v$, $f(x_2) = w_2$.

(4′) For $0 \leq x_2 < w_1$, $f(x_2) = w_2$.

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